

3-4

Solving Multi-Step Inequalities



Vocabulary

Review

- Multiple Choice You must *reverse* the direction of an inequality symbol when you multiply both sides of an inequality by a number that is ?.
 - A less than 0
 - B greater than 0
 - C less than or equal to 0
 - D greater than or equal to 0
- Write *reverse* if the inequality symbol will change when you solve the inequality. Write *same* if the symbol will remain the same.

$3t < 6$	$-8s < 4$	$5x \leq -10$	$-4 \leq -2a$
<u>same</u>	<u>reverse</u>	<u>same</u>	<u>reverse</u>

Vocabulary Builder



at least (adverbial phrase) **at least;** **at most** (adverbial phrase) **at most**

Main Idea: The phrase at least describes the least possible number that can be used. The phrase at most describes the greatest possible number that can be used.

Using Symbols: The inequality $x \geq 5$ means "x is at least 5." The inequality $x \leq 5$ means "x is at most 5."

Use Your Vocabulary

- Complete each sentence with the words *at least* or *at most*.
 - You must be ? 18 years of age to vote in a national election. _____
 - An elevator can safely carry ? 15 people. _____
 - When water boils, you know the temperature is ? 212°F. _____
 - If all books cost \$3 and Jane has \$20, she can buy ? 6 books. _____
- Use your answers to Exercise 3. Complete each inequality with \leq or \geq .

y	18	e	15	w	212	$3b$	20
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Problem 1 Using More Than One Step

Got It? What are the solutions of the inequality $-6a - 7 \leq 17$?
Check your solutions.

5. Circle the first step in solving the inequality. Then underline the second step.

1st

Add 7 to each side.

Divide each side by 6.

2nd

Divide each side by -6
and reverse the inequality.

Subtract 7 from each side.

Multiply each side by 6.

Multiply each side by -6
and reverse the inequality.

6. Use your answers to Exercise 5 to solve the inequality.

$$\begin{aligned} -6a - 7 &\leq 17 \\ -6a - 7 + 7 &\leq 17 + 7 \\ -6a &\leq 24 \\ \frac{-6a}{-6} &\leq \frac{24}{-6} \\ a &\geq -4 \end{aligned}$$

7. Check the endpoint by substituting into the related equation, $-6a - 7 = 17$.

8. Check the inequality symbol by substituting into the original inequality, $-6a - 7 \leq 17$.

$$\begin{aligned} -6(0) - 7 &\leq 17 \\ -7 &\leq 17 \quad \checkmark \end{aligned}$$

Problem 2 Writing and Solving a Multi-Step Inequality

Got It? You want to make a rectangular banner that is 18 ft long. You have no more than 48 ft of trim for the banner. What are the possible widths of the banner?



18 ft

9. Circle the formula for the perimeter of a rectangle.

$C = 2\pi r$

$A = \ell w$

$d = rt$

$P = 2\ell + 2w$

10. Write an algebraic expression to describe the distance around a rectangular banner with a length of 18 ft and a width of w ft.

$$P \geq 2 \cdot (18) + 2w$$

11. The distance around the banner should be at least / at most 48 feet.

12. Use the expression you wrote in Exercise 10 and the information from Exercise 11. Write an inequality to represent the situation described in the problem. Then solve your inequality.

$$\begin{aligned} 48 &\geq 36 + 2w \\ 36 + 2w &\leq 48 \\ 36 - 36 + 2w &\leq 48 - 36 \end{aligned}$$

$$\frac{2w}{2} \leq \frac{12}{2}$$

$$w \leq 6 \text{ ft}$$

13. The width of the banner should be at most _____ feet.

Problem 3 Using the Distributive Property

Got It? What are the solutions of $15 \leq 5 - 2(4m + 7)$? Check your solutions.

14. Use the justifications at the right to solve the inequality.

$$\begin{array}{ll}
 15 \leq 5 - 2(4m + 7) & \text{Write the original inequality.} \\
 15 \leq 5 - 8m - 14 & \text{Distributive Property} \\
 15 \leq -8m - 9 & \text{Subtract.} \\
 15 + 9 \leq -8m - 9 + 9 & \text{Addition Property of Inequality} \\
 24 \leq -8m & \text{Add.} \\
 \frac{24}{-8} \geq \frac{-8m}{-8} & \text{Division Property of Inequality} \\
 -3 \geq m & \text{Simplify.} \\
 \boxed{m \leq -3} &
 \end{array}$$

$$\begin{array}{r}
 5 + -14 \\
 -9
 \end{array}$$

15. Check your solutions by following the steps below.

$$\begin{array}{ll}
 15 \stackrel{?}{\leq} 5 - 2(4 \cdot \boxed{} + 7) & \text{Substitute one of your solutions to Exercise 14.} \\
 15 \stackrel{?}{\leq} 5 - 2(\boxed{} + 7) & \text{Multiply within parentheses.} \\
 15 \stackrel{?}{\leq} 5 - 2 \cdot \boxed{} & \text{Add within parentheses.} \\
 15 \stackrel{?}{\leq} 5 - \boxed{} & \text{Multiply.} \\
 15 \leq \boxed{} & \text{Simplify.}
 \end{array}$$

Problem 4 Solving an Inequality With Variables on Both Sides

Got It? What are the solutions of $3b + 12 > 27 - 2b$? Check your solutions.

16. The inequality is solved below. Write a justification for each step.

$$\begin{array}{ll}
 3b + 12 > 27 - 2b & \text{Rewrite the inequality} \\
 2b + 3b + 12 > 27 - 2b + 2b & \text{Addition Property of Inequality} \\
 \underline{5b + 12} > \underline{27} & \text{Simplify} \\
 5b + 12 - 12 > 27 - 12 & \text{Subtraction Property of Inequality} \\
 \rightarrow \frac{5b}{5} > \frac{15}{5} & \text{Division Property of Inequality} \\
 b > 3 & \text{Simplify.}
 \end{array}$$

17. Check your solutions in the original inequality.

18. Are your solutions correct?

Yes / No

Problem 5 Inequalities With Special Solutions

Got It? What are the solutions of the inequality $9 + 5n \leq 5n - 1$?

19. Solve the inequality $9 + 5n \leq 5n - 1$.

$$9 + \cancel{5n} - \cancel{5n} \leq \cancel{5n} - \cancel{5n} - 1$$

$$9 \leq -1$$

20. The inequality $9 + 5n \leq 5n - 1$ is always never true.

So, the solution is all real numbers there is no solution!

Always true =

$$9 \leq 10$$

Lesson Check • Do you UNDERSTAND?

Error Analysis Your friend says that the solutions of the inequality $-2(3 - x) > 2x - 6$ are all real numbers. Do you agree with your friend? Explain. What if the inequality symbol were \geq ?

21. The inequality $-2(3 - x) > 2x - 6$ is solved below. Write a justification from the box for each step.

$$-2(3 - x) > 2x - 6$$

$$-6 + 2x > 2x - 6$$

$$-6 + 2x - 2x > 2x - 6 - 2x$$

$$-6 > -6$$

Distributive Property
Simplify.
Subtraction Property
of Inequality
Write the original
inequality.

22. Look at the final inequality in Exercise 21. Is the inequality ever true?

Yes / No

23. Do you agree with your friend? Explain. What if the inequality symbol were \geq ?

Math Success

Check off the vocabulary words that you understand.

multi-step inequalities

properties of inequality

solutions

Rate how well you can solve multi-step inequalities.

